

Year 10 Mathematics Worked Solutions

Middle School Exams 2026

Solutions follow the practice booklet question numbers. Mathematical expressions, fractions and powers use editable Word equations. Supplied diagrams are schematic unless the question asks for an accurate drawing. Retain unrounded intermediate values. For graphical answers, sensible plotting and measuring tolerances are acceptable.

Number Skills

Question 1

- (a) Multiply first: $-14 - 18 + 8 = -24$.
(b) $2.8 \times 1.5 = 4.2$; $7.35 - 4.2 = 3.15$.
(c) $8 \div 0.32 = 25$.

Question 2

- (a) $29 = 4 \times 6 + 5$, so $4 \frac{5}{6}$; $\frac{3 \times 8 + 5}{8} = \frac{29}{8}$.
(b) $\frac{14}{6} + \frac{11}{6} = \frac{25}{6} = 4 \frac{1}{6}$.
(c) $\frac{84}{20} - \frac{55}{20} = \frac{29}{20} = 1 \frac{9}{20}$.
(d) $(\frac{8}{5})(\frac{9}{4}) = \frac{72}{20} = \frac{18}{5} = 3 \frac{3}{5}$.
(e) $(\frac{7}{2}) \div (\frac{7}{6}) = (\frac{7}{2})(\frac{6}{7}) = 3$.

Question 3

- (a) $(\frac{3}{7}) \times 84 = 36$ students.
(b) $36 \div 6 = 6$ groups.
(c) $6 \times 2.5 = 15$ litres.

Question 4

- (a) $120 : 45 = 8 : 3$.
(b) Multiply both parts by 5: missing value 45.
(c) 13 parts; one part = $\frac{936}{13} = 72$. Shares \$360 and \$576.
(d) 9 parts; syrup = $(\frac{2}{9}) \times 2.7 = 0.6$ litres.

Question 5

- (a) $\frac{252}{3.5} = 72$ pages per minute.
(b) $\frac{156}{12} = 13$ km per litre.
(c) Fuel = $\frac{390}{13} = 30$ litres. Cost = $30 \times 2.65 = \$79.50$.

Question 6

(a) 7.24×10^{-5} .

(b) 608000.

(c) $(3.6 \times 2.5) \times 10^4 = 9 \times 10^4$.

(d) $\left(\frac{7.2}{4.8}\right) \times 10^5 = 150000$ files.

(e) $360 = 36 \times 10 = 2^3 \times 3^2 \times 5$.

Percentages

Question 7

(a) $(\frac{7}{16}) \times 100 = 43.75\%$; $0.082 \times 100 = 8.2\%$.

(b) $0.18 \times 650 = 117$.

(c) $(\frac{27}{36}) \times 100 = 75\%$.

Question 8

(a) $64 \times 1.35 = \$86.40$.

(b) $86.40 \times 1.15 = \$99.36$.

(c) $99.36 \times 0.80 = \$79.488$, or $\$79.49$.

(d) Multipliers give $1.20 \times 0.80 = 0.96$, so the result is 4% below the original.

Question 9

(a) 85% represents \$357: original = $\frac{357}{0.85} = \$420$.

(b) Before $GST = \frac{529}{1.15} = \$460$. $GST = 529 - 460 = \$69$.

Question 10

(a) $I = Prt = 1250 \times 0.042 \times 4 = \$210$. Balance = $\$1460$.

(b) $A = 1250(1.04)^4 = \$1462.3232$, or $\$1462.32$.

(c) Compound option is larger by $1462.3232 - 1460 = \$2.32$.

Question 11

(a) Listed total = $48 \times 32 = \$1536$. Discounted total = $1536 \times 0.875 = \$1344$. With $GST = 1344 \times 1.15 = \$1545.60$. Invoice = $\$1563.60$.

(b) Income = $48 \times 34 = \$1632$. Surplus = $1632 - 1563.60 = \$68.40$.

Algebra One and Sequences

Question 12

- (a) Three x factors and four x factors give seven: x^7 .
(b) Cancel four a factors, leaving a^5 .
(c) $b^2 \times b^2 \times b^2$ gives six b factors: b^6 .
(d) $3p^4q^2; 8r^9s^6$. Division subtracts powers; a power of a product applies to every factor.

Question 13

- (a) $13a + 6b - 4$.
(b) $6x^2 - 15x - x^2 - 4x = 5x^2 - 19x$.
(c) $2(9) - 3(-3)(2) + 4 = 18 + 18 + 4 = 40$.
(d) $(\frac{5}{9})(77 - 32) = (\frac{5}{9})45 = 25$.

Question 14

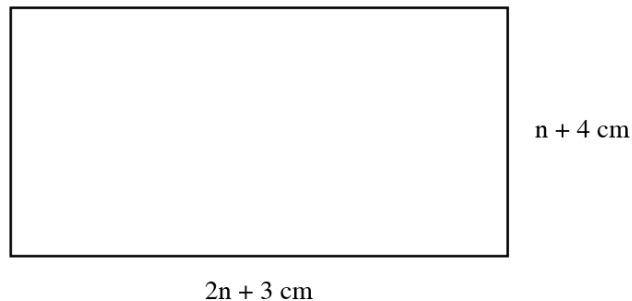
- (a) Four adjoining squares in a row use 13 sticks.
(b) First square needs 4; each additional square shares one side and adds 3. $M = 4 + 3(n - 1) = 3n + 1$.
(c) $3(28) + 1 = 85$.
(d) $3n + 1 = 100$ gives $n = 33$, so yes.

Question 15

- (a) Differences 3, 5, 7, 9 show $n^2 + 2$. Next terms 38, 51.
(b) Each term is $n^3 + 1$. Next terms 217, 344.
(c) Difference 6; $T_n = 6n + 8$. Next terms 38, 44.

Question 16

- (a) $P = 2[(2n + 3) + (n + 4)] = 6n + 14$ cm.
(b) $P = 36 + 14 = 50$ cm.
(c) $A = (2n + 3)(n + 4) = 2n^2 + 11n + 12$ cm^2 , so quadratic.



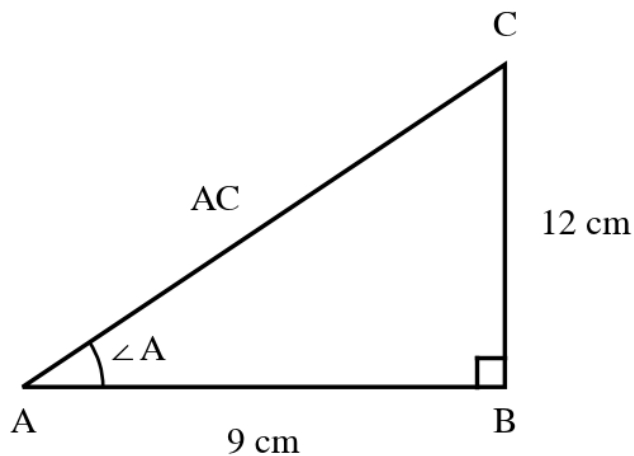
Pythagoras and Trigonometry

Question 17

(a) Opposite to A : BC ; adjacent: AB ; hypotenuse: AC .

(b) $AC^2 = 9^2 + 12^2 = 225$, so $AC = 15$ cm.

(c) $\tan A = \frac{12}{9}$. $A = \tan^{-1}(\frac{12}{9}) = 53.1^\circ$.



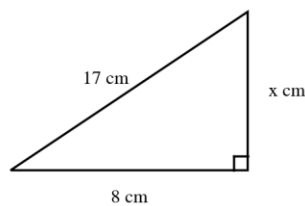
Question 18

(a) $x^2 = 17^2 - 8^2 = 225$, so $x = 15$ cm.

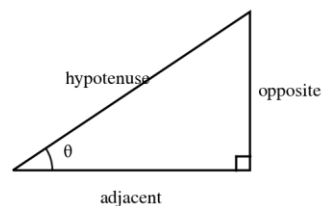
(b) In degree mode: 0.602, 0.469, 0.445 respectively.

(c) $\theta = \sin^{-1}(0.428) = 25.3^\circ$.

(a)



(c) $\sin \theta = 0.428$



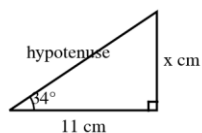
Question 19

(a) $\tan 34^\circ = \frac{x}{11}$, so $x = 11 \tan 34^\circ = 7.42$ cm.

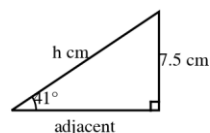
(b) $\sin 41^\circ = \frac{7.5}{h}$, so $h = \frac{7.5}{\sin 41^\circ} = 11.4$ cm.

(c) $\cos 28^\circ = \frac{a}{19}$, so $a = 19 \cos 28^\circ = 16.8$ cm.

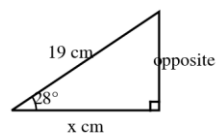
(a)



(b)



(c)

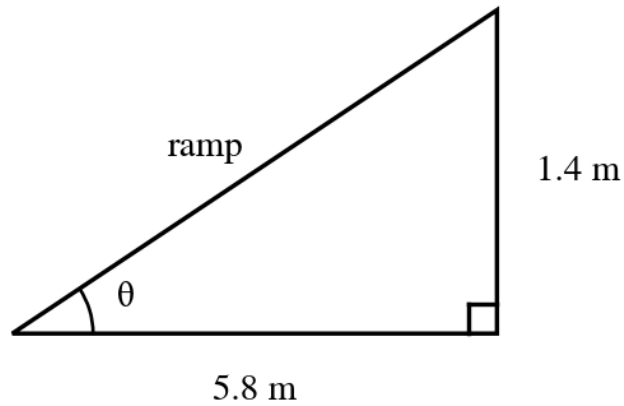


Question 20

(a) $L = \sqrt{5.8^2 + 1.4^2} = \sqrt{35.6} = 5.966... \approx 5.97 \text{ m.}$

(b) $\theta = \tan^{-1}\left(\frac{1.4}{5.8}\right) = 13.570... \approx 13.6^\circ.$

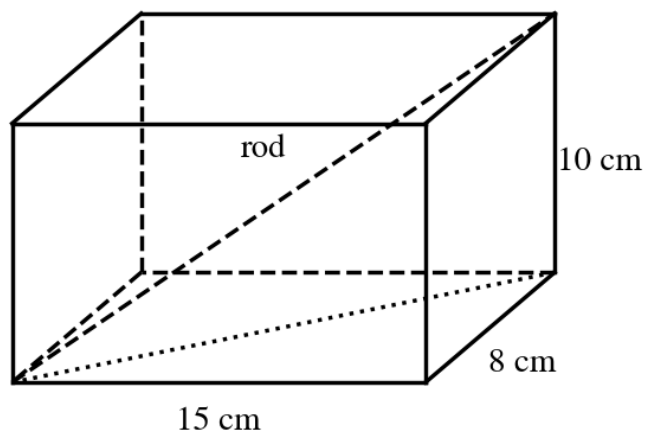
(c) No: $13.570...^\circ > 12^\circ.$

**Question 21**

(a) $d = \sqrt{15^2 + 8^2} = \sqrt{289} = 17 \text{ cm.}$

(b) $L = \sqrt{17^2 + 10^2} = \sqrt{389} = 19.7 \text{ cm to 3 significant figures.}$

(c) The rod projects onto the base diagonal. $\tan \theta = \frac{10}{17}$; $\theta = 30.5^\circ.$



Mensuration

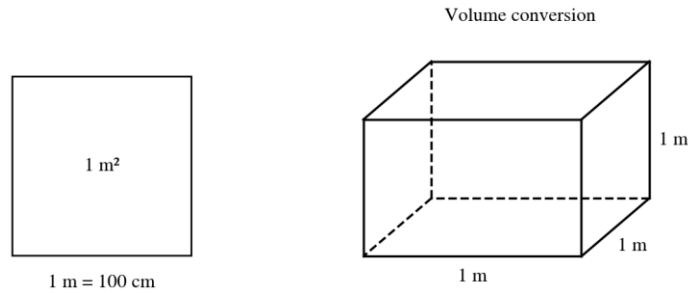
Question 22

(a) 3750 m; 68 cm.

(b) $2.4 \times 10000 = 24000 \text{ cm}^2$; $\frac{85000}{100} = 850 \text{ cm}^2$.

(c) $0.032 \times 1000000 = 32000 \text{ cm}^3$; $\frac{4500}{1000} = 4.5$ litres.

(d) $8.55 \leq L < 8.65$ cm; the upper bound is excluded.



Question 23

(a) $13 \times 7 = 91 \text{ cm}^2$.

(b) $\frac{1}{2} \times 14 \times 9 = 63 \text{ cm}^2$.

(c) $11 \times 6 = 66 \text{ cm}^2$.

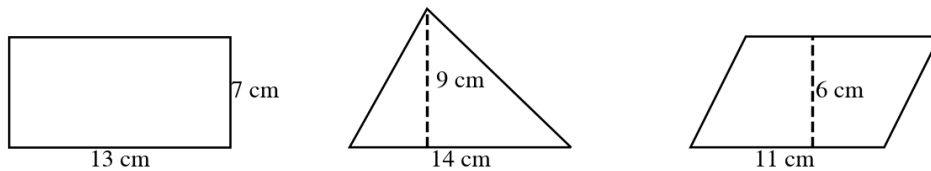
(d) $\frac{1}{2}(9 + 17)5 = 65 \text{ cm}^2$.

(e) Area = $63 - 6 = 57 \text{ m}^2$. Removing a corner replaces two outer lengths by two equal inner lengths, so perimeter = $2(9 + 7) = 32$ m.

(a)

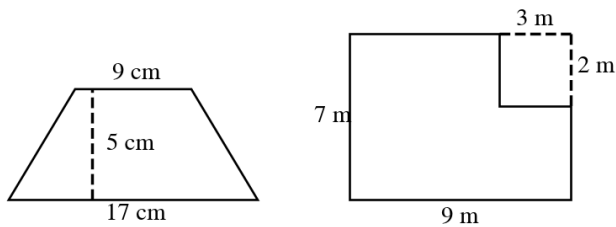
(b)

(c)



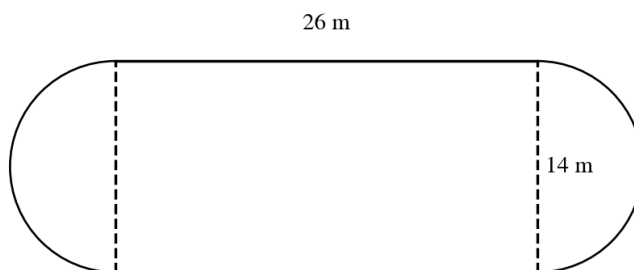
(d)

(e)



Question 24

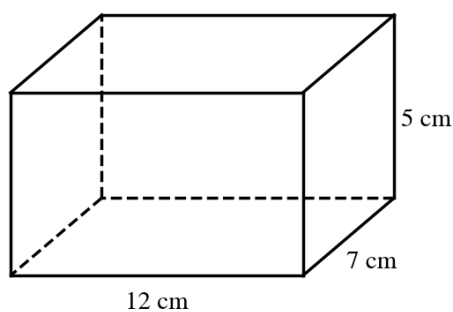
- (a) Two straight lengths and two semicircles: $P = 52 + 14\pi = 95.98... \approx 96.0$ m.
 (b) $A = 26 \times 14 + \pi(7^2) = 364 + 49\pi = 517.94... \approx 518$ m².
 (c) $517.94.../85 = 6.09...$ so 7 bags.



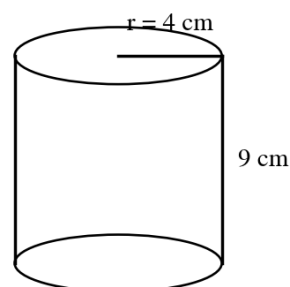
Question 25

- (a) $2(12 \times 7 + 12 \times 5 + 7 \times 5) = 358$ cm².
 (b) $2\pi r^2 + 2\pi rh = 32\pi + 72\pi = 104\pi \approx 327$ cm².
 (c) $4\pi r^2 = 144\pi \approx 452$ cm².
 (d) One base and curved side: $\pi(15^2) + 2\pi(15)(28) = 1065\pi \approx 3350$ cm².

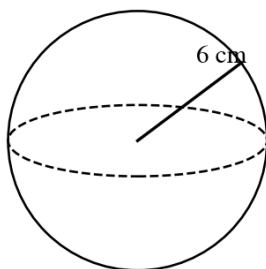
(a) Closed cuboid



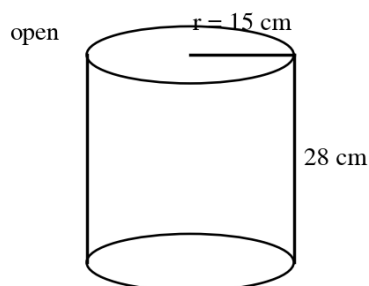
(b) Closed cylinder



(c) Sphere



(d) Bucket



Question 26

(a) $8 \times 6 \times 11 = 528 \text{ cm}^3$.

(b) $(\frac{1}{2} \times 7 \times 4) \times 18 = 252 \text{ cm}^3$.

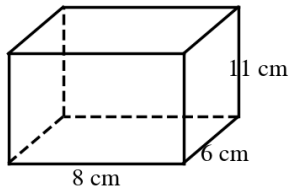
(c) $\pi(3^2)14 = 126\pi \text{ cm}^3$.

(d) $(\frac{1}{3})\pi(5^2)12 = 100\pi \text{ cm}^3$.

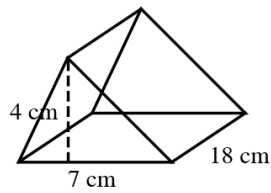
(e) $(\frac{1}{3})(9 \times 8)10 = 240 \text{ cm}^3$.

(f) $(\frac{4}{3})\pi(3^3) = 36\pi \text{ cm}^3$.

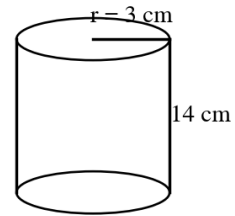
(a) Cuboid



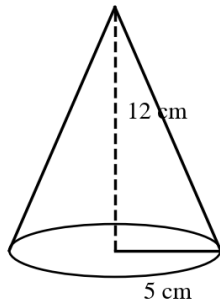
(b) Triangular prism



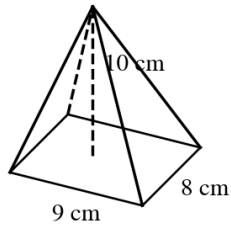
(c) Cylinder



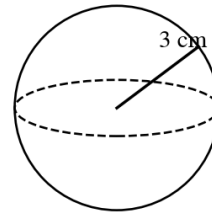
(d) Cone



(e) Pyramid



(f) Sphere

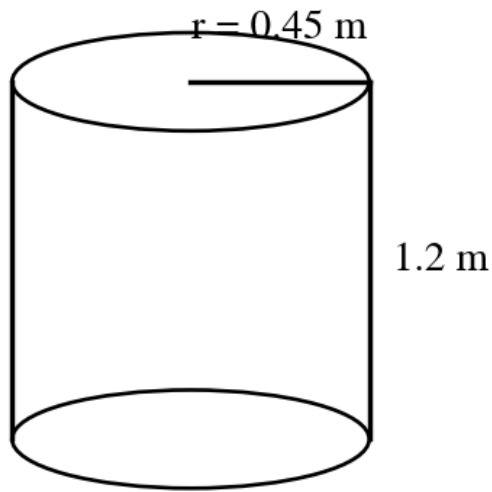


Question 27

(a) $V = \pi(0.45^2)(1.2) = 0.243\pi \text{ m}^3 = 243\pi \text{ litres} \approx 763 \text{ litres.}$

(b) $\text{Time} = 0.8 \times \frac{243\pi}{18} = 10.8\pi \approx 33.9 \text{ minutes.}$

(c) $44.5 \leq r < 45.5 \text{ cm.}$ Capacity lower bound $= \pi(44.5^2) \frac{120}{1000} = 237.63\pi \approx 746.54 \text{ litres.}$ Upper bound $= \pi(45.5^2) \frac{120}{1000} = 248.43\pi \approx 780.46 \text{ litres; excluded.}$



Linear Equations and Inequalities

Question 28

- (a) $12x - 8 = 5x + 27$; $7x = 35$; $x = 5$.
(b) Multiply by 10: $4y - 2 + 5y = 70$; $9y = 72$; $y = 8$.
(c) $-2t \leq 12$; divide by -2 and reverse the sign: $t \geq -6$. Filled dot at -6 ; arrow to the right.

Question 29

- (a) $R = \frac{V}{I}$, $I \neq 0$.
(b) $3y = 5x + 7$; $x = \frac{3y-7}{5}$.
(c) $2A = bh$; $h = \frac{2A}{b}$, $b \neq 0$.
(d) $z^2 = 3w + 4$; $w = \frac{z^2-4}{3}$. The original equation requires $z \geq 0$.

Question 30

- (a) $y = 19 - 3x$. Substitute: $2x - 3(19 - 3x) = -2$; $11x = 55$, $x = 5$; $y = 4$.
(b) $b = 2a - 7$. Substitute: $4a + 3(2a - 7) = 29$; $10a = 50$, $a = 5$; $b = 3$.

Question 31

- (a) $C = 24 + 9h$: fixed fee plus 9 dollars for every hour.
(b) $24 + 9h \leq 87$; $9h \leq 63$; $h \leq 7$, with $h \geq 0$.
(c) 7 whole hours.

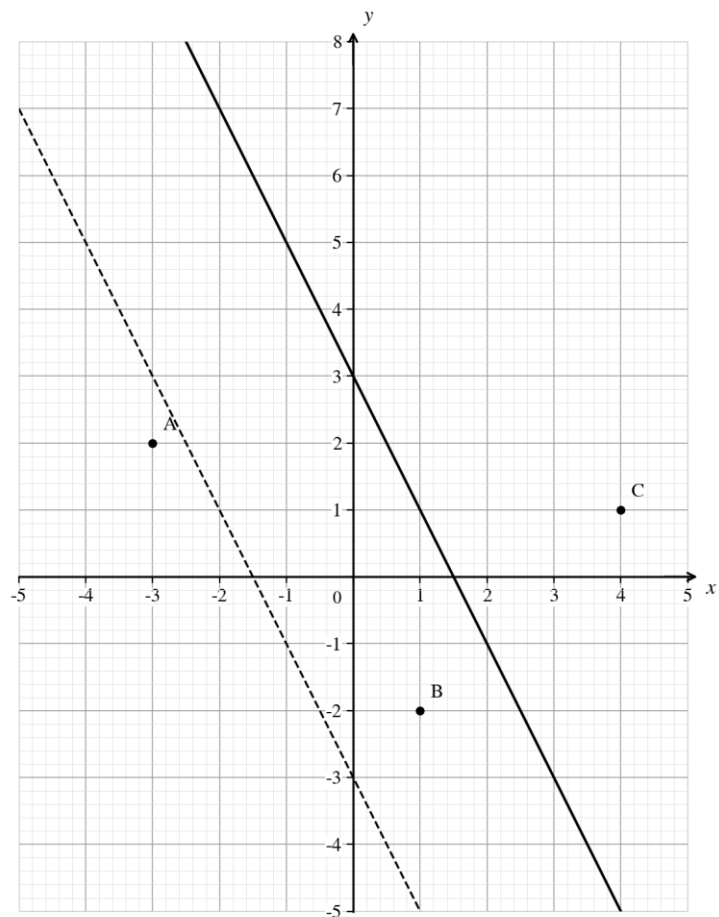
Question 32

- (a) Let a be adult tickets and s student tickets: $a + s = 73$; $14a + 9s = 817$.
(b) Subtract $9(a + s) = 657$ from the income equation: $5a = 160$; $a = 32$; $s = 41$.
(c) $32 + 41 = 73$; $14(32) + 9(41) = 448 + 369 = 817$.

Straight Line Graphs

Question 33

- (a) See the plotted points.
- (b) y values are 7, 5, 3, 1, -1 , -3 ; join with a straight line.
- (c) Gradient -2 ; y intercept 3, at $(0, 3)$.
- (d) Same gradient -2 with intercept -3 : $y = -2x - 3$.



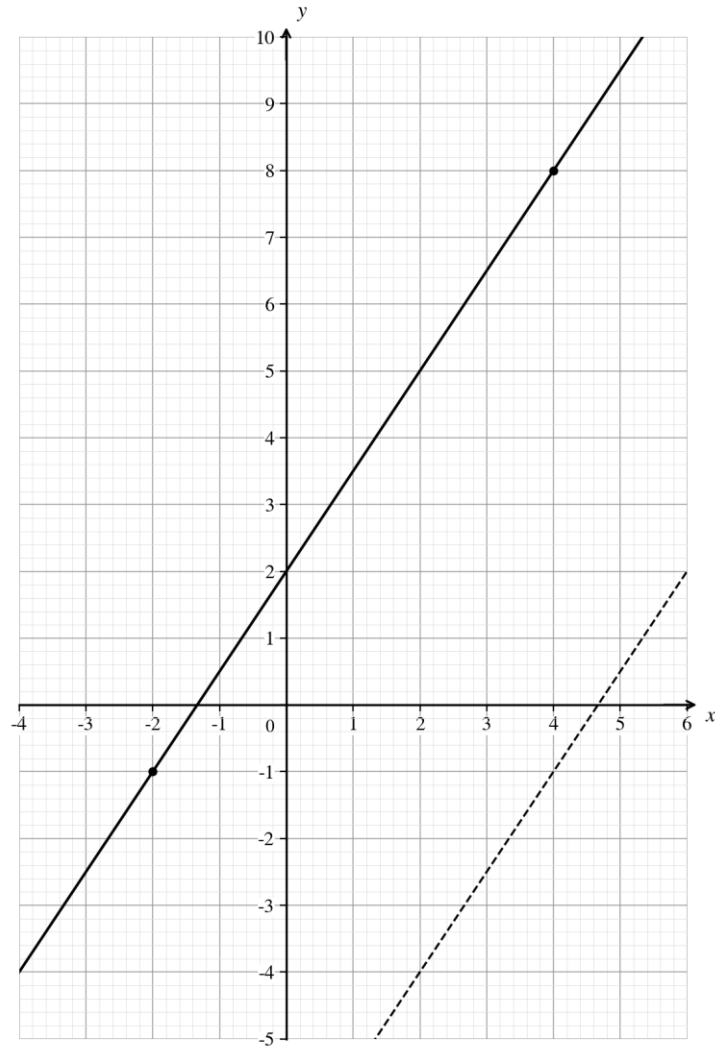
Question 34

(a) Intercept 2.

(b) $m = \frac{8 - (-1)}{4 - (-2)} = \frac{9}{6} = 1.5; y = 1.5x + 2.$

(c) $-4 = 1.5(2) + c$, so $c = -7; y = 1.5x - 7.$

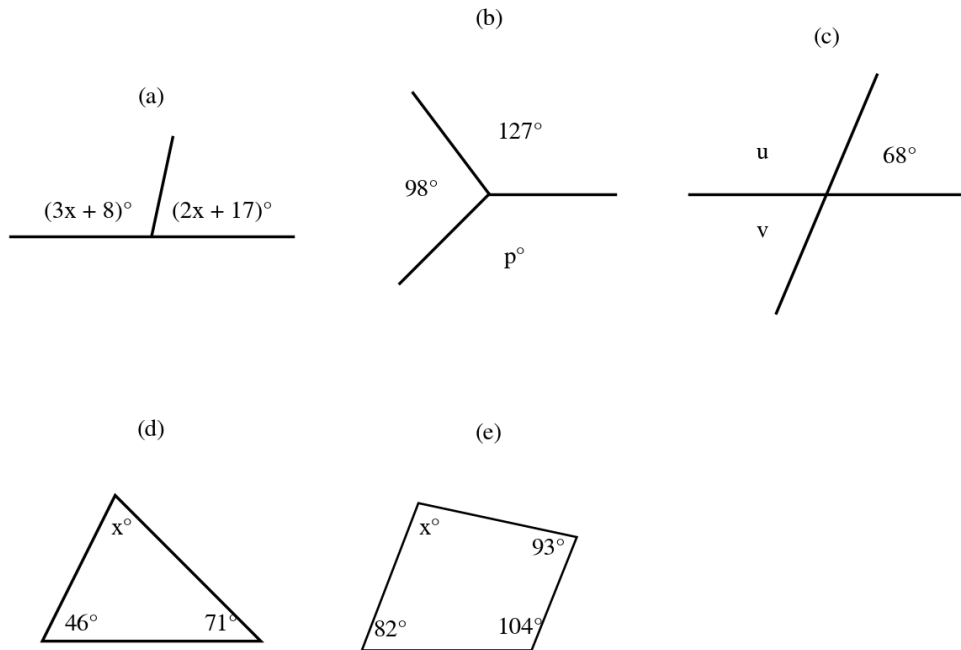
(d) At $x = 10, y = 15 + 2 = 17$, so yes.



Geometry

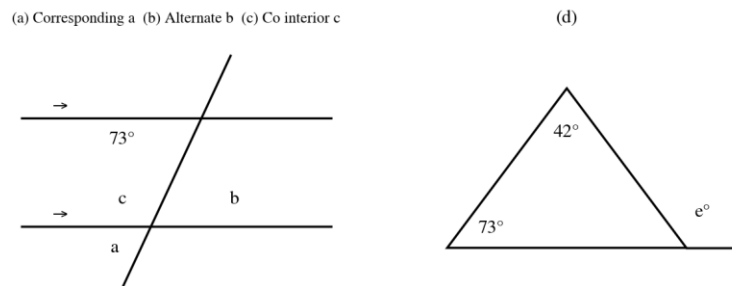
Question 35

- (a) $5x + 25 = 180$, so $x = 31$; angles 101° and 79° (straight line).
 (b) $p = 360 - 127 - 98 = 135^\circ$ (angles around a point).
 (c) Vertically opposite 68° ; adjacent $= 180 - 68 = 112^\circ$.
 (d) $180 - 46 - 71 = 63^\circ$ (triangle).
 (e) $360 - 82 - 104 - 93 = 81^\circ$ (quadrilateral).



Question 36

- (a) 73° , corresponding angles are equal.
 (b) 73° , alternate interior angles are equal.
 (c) $180 - 73 = 107^\circ$, co interior angles sum to 180° .
 (d) Third angle $= 180 - 73 - 42 = 65^\circ$. Exterior $= 180 - 65 = 115^\circ$; equivalently the two opposite interior angles sum to 115° .



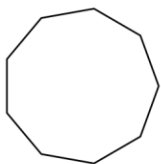
Question 37

(a) $(9 - 2) \times 180 = 1260^\circ$.

(b) $n = \frac{360}{24} = 15$; interior angle $= 180 - 24 = 156^\circ$.

(c) Hexagon sum $= (6 - 2)180 = 720^\circ$. Given sum 594° ; missing angle 126° .

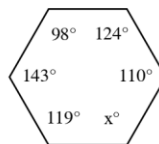
(a) Nonagon



(b) Part of a regular polygon



(c) Hexagon



Question 38

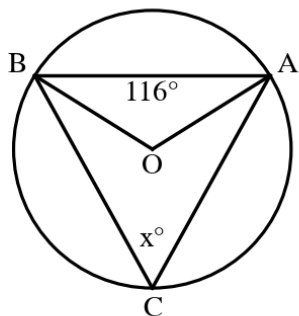
(a) 58° : angle at the centre is twice the angle at the circumference standing on the same minor arc.

(b) 39° : angles in the same segment, subtending chord AB , are equal.

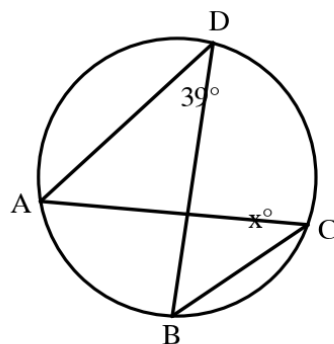
(c) 90° : angle in a semicircle.

(d) $\angle R = 180 - 112 = 68^\circ$; $\angle S = 180 - 87 = 93^\circ$, opposite angles of a cyclic quadrilateral are supplementary.

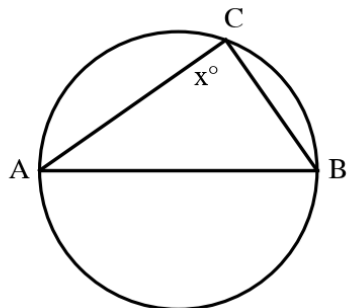
(a)



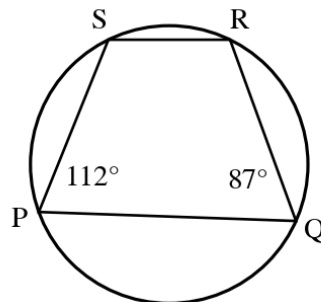
(b)



(c)



(d)

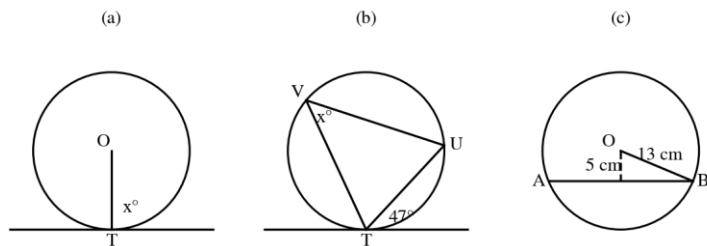


Question 39

(a) 90° : tangent perpendicular to radius at contact.

(b) $\angle TVU = 47^\circ$, alternate segment theorem.

(c) Perpendicular from the centre bisects the chord. Half chord = $\sqrt{13^2 - 5^2} = 12$ cm. $AB = 24$ cm.

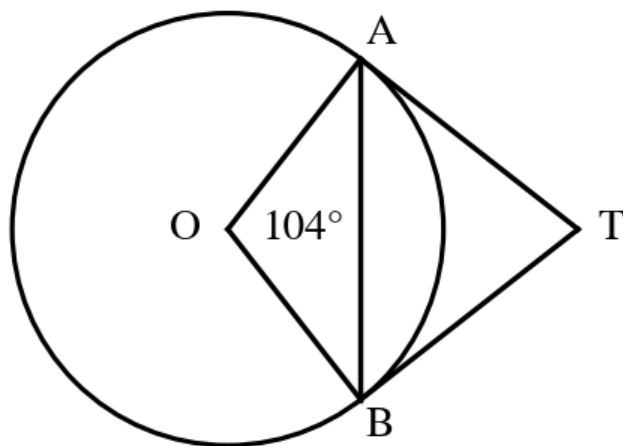


Question 40

(a) Both 90° , radius perpendicular to tangent.

(b) Quadrilateral $OATB$ angle sum 360° : $\angle ATB = 360 - 104 - 90 - 90 = 76^\circ$.

(c) $OA = OB$ (radii), so triangle is isosceles. Each base angle = $\frac{180 - 104}{2} = 38^\circ$.



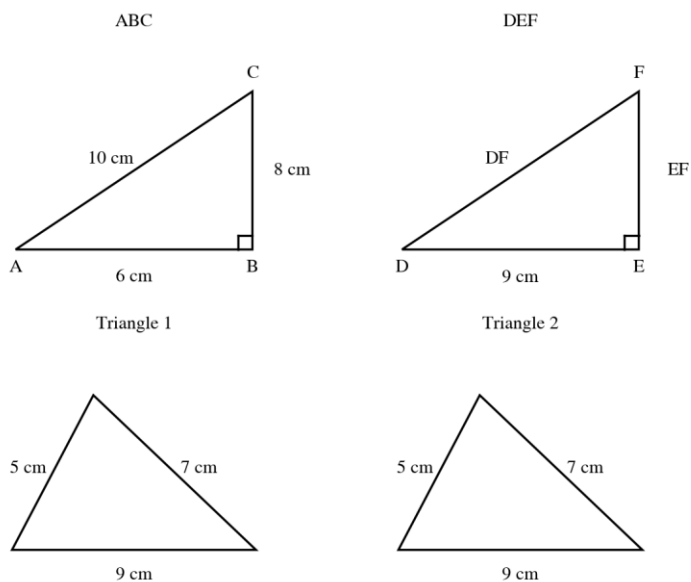
Question 41

(a) $\frac{9}{6} = 1.5$.

(b) $EF = 8 \times 1.5 = 12 \text{ cm}$; $DF = 10 \times 1.5 = 15 \text{ cm}$.

(c) Area factor = $1.5^2 = 2.25$; area = $24 \times 2.25 = 54 \text{ cm}^2$.

(d) Congruent by SSS, with matching sides paired regardless of listing order.



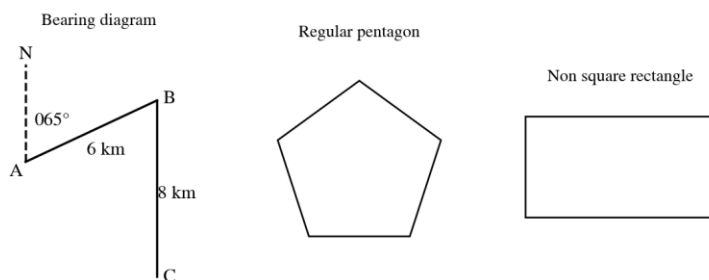
Question 42

(a) $065^\circ + 180^\circ = 245^\circ$.

(b) $AB = 3 \text{ cm}$; measure 65° clockwise from north.

(c) $BC = 4 \text{ cm}$ due south. Correct drawing gives $AC \approx 7.71 \text{ km}$ (about 3.86 cm on paper); allow sensible ruler tolerance.

(d) Regular pentagon: 5 lines, rotational order 5. Non square rectangle: 2 lines, rotational order 2.



Question 43

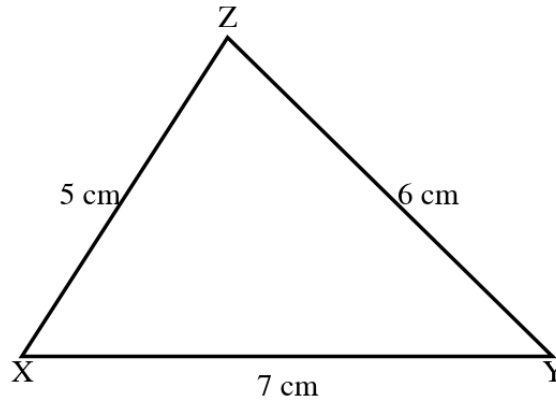
(a) Draw $XY = 7$ cm. Draw radius 5 cm arc centred X and radius 6 cm arc centred Y . Label an intersection Z ; join XZ and YZ . The reflected triangle is also valid.

(b) With equal compass radius greater than 3.5 cm, draw intersecting arcs above and below XY from X and Y . Join their intersections.

(c) $XM = \frac{7}{2} = 3.5$ cm.

(d) A point P on the bisector forms two right triangles with PM common and $XM = YM$; they are congruent by SAS , so $PX = PY$.

Reference sketch only construct your own triangle below

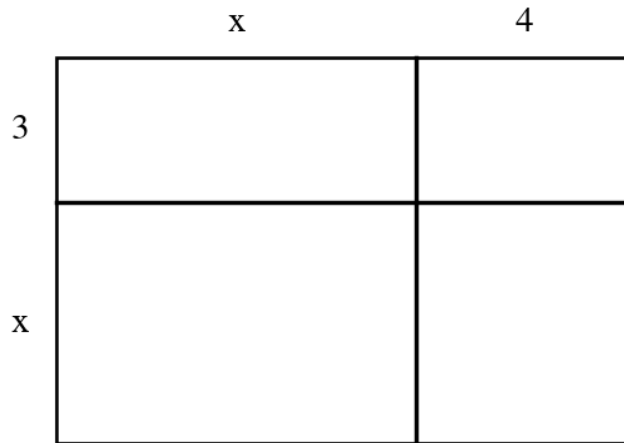


Algebra Two

Question 44

- (a) $-8x + 28$.
(b) $x^2 - 2x + 6x - 12 = x^2 + 4x - 12$.
(c) $6t^2 + 15t - 4t - 10 = 6t^2 + 11t - 10$.
(d) Split sides into x and 4, and x and 3. Four areas are x^2 , $3x$, $4x$ and 12. Total $x^2 + 7x + 12$.

(d) Area model



Question 45

- (a) $5a^2b(3a - 4b)$.
(b) $(7m - 4)(7m + 4)$.
(c) Product -40 and sum 3: 8 and -5 ; $(x + 8)(x - 5)$.
(d) Split $7y$ as $6y + y$: $(2y + 1)(y + 3)$.
(e) $3z(z^2 - 9) = 3z(z - 3)(z + 3)$.

Question 46

- (a) $\frac{2x}{3y}$; $x \neq 0$ and $y \neq 0$ in the original expression.
(b) $[(t - 5)(t + 5)] \frac{1}{(t + 5)(t - 3)} = \frac{t - 5}{t - 3}$; $t \neq -5, 3$.
(c) $\frac{2k + 1 + 2(k - 4)}{6} = \frac{4k - 7}{6}$. No variable restrictions.

Question 47

- (a) $(x + 7)(x - 5) = 0$, so $x = -7$ or 5 .
(b) $u^2 = 25$, so $u = -5$ or 5 .
(c) $(2v + 1)(v - 3) = 0$, so $v = -\frac{1}{2}$ or 3 .
(d) $w^2 - 6w - 16 = 0$; $(w - 8)(w + 2) = 0$; $w = 8$ or -2 .

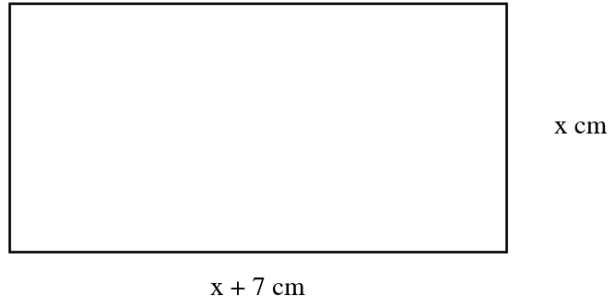
Question 48

(a) $x(x + 7) = 120$; $x^2 + 7x - 120 = 0$.

(b) $(x + 15)(x - 8) = 0$; $x = 8$ or -15 . Reject -15 because a width is positive.

(c) Length = 15 cm; $P = 2(8 + 15) = 46$ cm.

(d) $46 \times 0.08 = \$3.68$.



Vectors

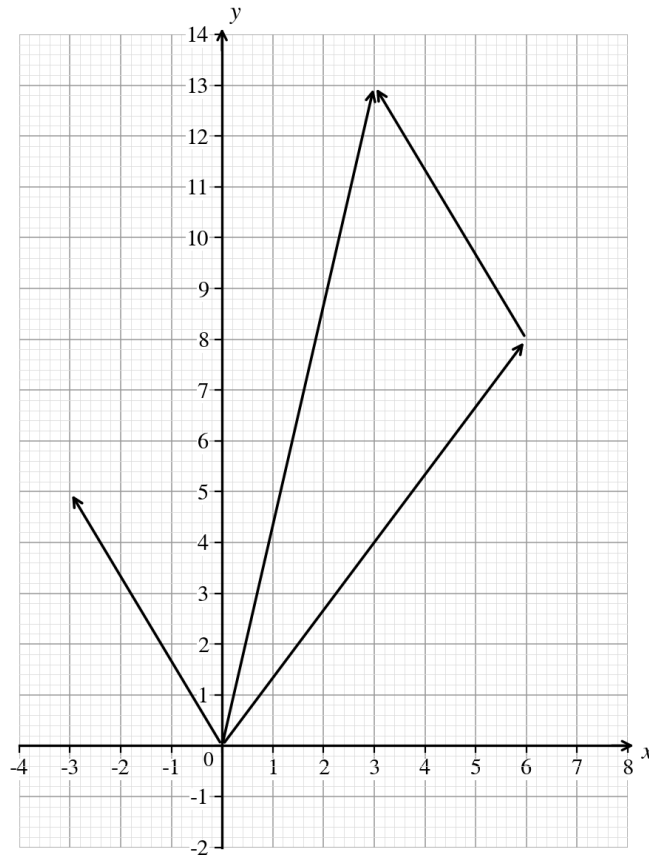
Question 49

(a) $a = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$; $b = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$. a means 6 units right and 8 units up.

(b) $3a = \begin{pmatrix} 18 \\ 24 \end{pmatrix}$; $a + b = \begin{pmatrix} 3 \\ 13 \end{pmatrix}$; $a - b = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$; $2a - 3b = \begin{pmatrix} 21 \\ 1 \end{pmatrix}$.

(c) $|a| = \sqrt{36 + 64} = 10$; direction = $\tan^{-1}\left(\frac{8}{6}\right) = 53.1^\circ$.

(d) Draw $O \rightarrow (6, 8)$, then $(6, 8) \rightarrow (3, 13)$; resultant $O \rightarrow (3, 13)$. Draw b from O separately.



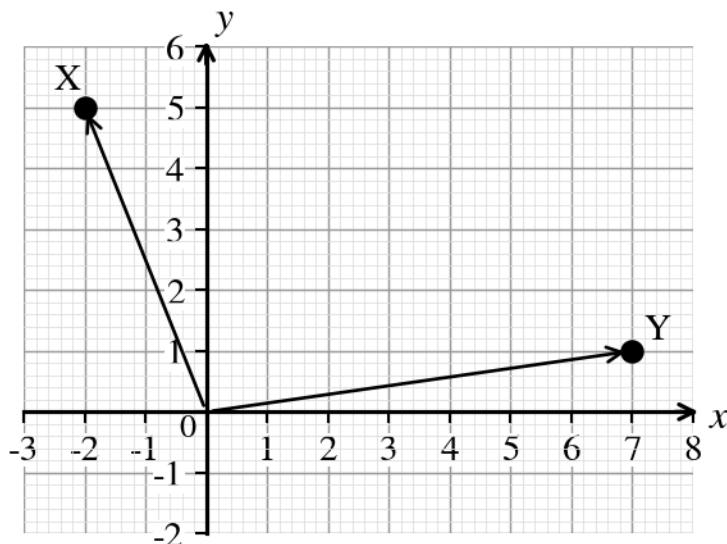
Question 50

(a) $OX = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$; $OY = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$; $XO = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$; $XY = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$.

(b) Draw arrows $X \rightarrow O \rightarrow Y$ and direct arrow $X \rightarrow Y$.

(c) $XY = \sqrt{9^2 + (-4)^2} = \sqrt{97} \approx 9.85$.

(d) $-2 + 7k = 19$, so $k = 3$. Second component $5 + 3(1) = 8$.



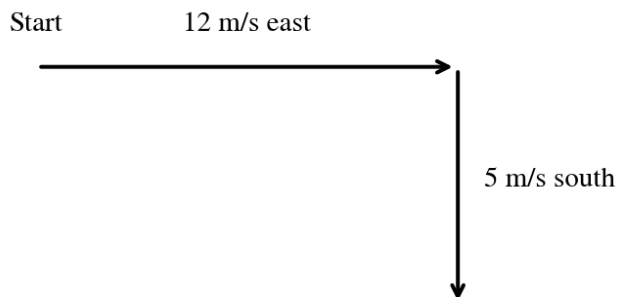
Question 51

(a) Drone $\begin{pmatrix} 12 \\ 0 \end{pmatrix}$; wind $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$; resultant $\begin{pmatrix} 12 \\ -5 \end{pmatrix} \frac{\text{m}}{\text{s}}$.

(b) Speed $= \sqrt{12^2 + 5^2} = 13 \frac{\text{m}}{\text{s}}$.

(c) Angle south of east $= \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$. Bearing $= 90 + 22.6 = 112.6^\circ$, or 113° to nearest degree.

(d) $\begin{pmatrix} 480 \\ -200 \end{pmatrix} \text{ m}$; magnitude $13 \times 40 = 520 \text{ m}$.



Transformations

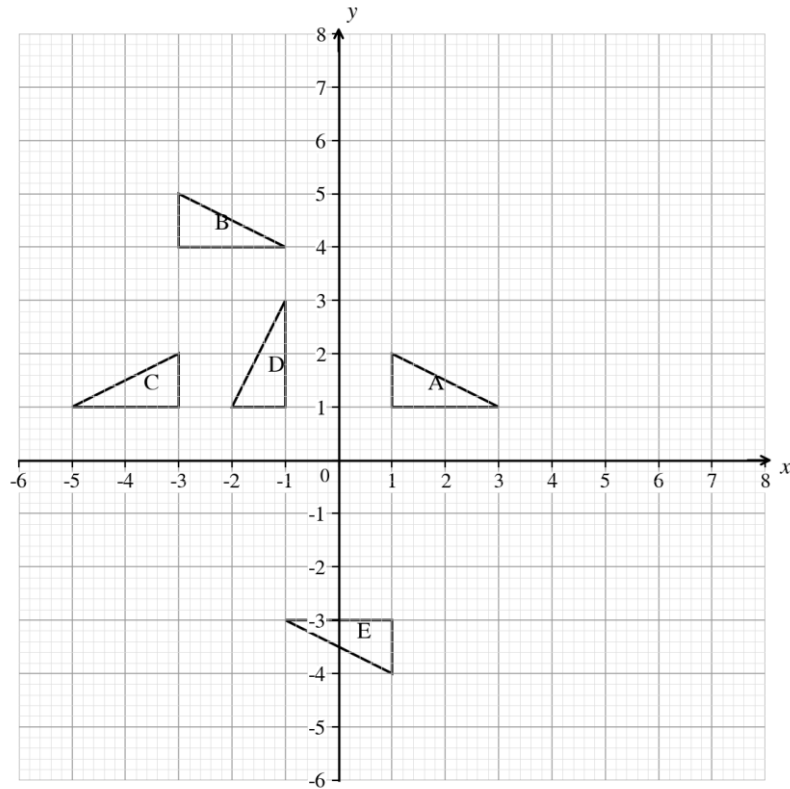
Question 52

(a) $B: (-3, 4), (-1, 4), (-3, 5)$.

(b) Reflection $x' = -2 - x$: $C: (-3, 1), (-5, 1), (-3, 2)$.

(c) $(x, y) \rightarrow (-y, x)$: $D: (-1, 1), (-1, 3), (-2, 1)$.

(d) $(x, y) \rightarrow (2 - x, -2 - y)$: $E: (1, -3), (-1, -3), (1, -4)$.



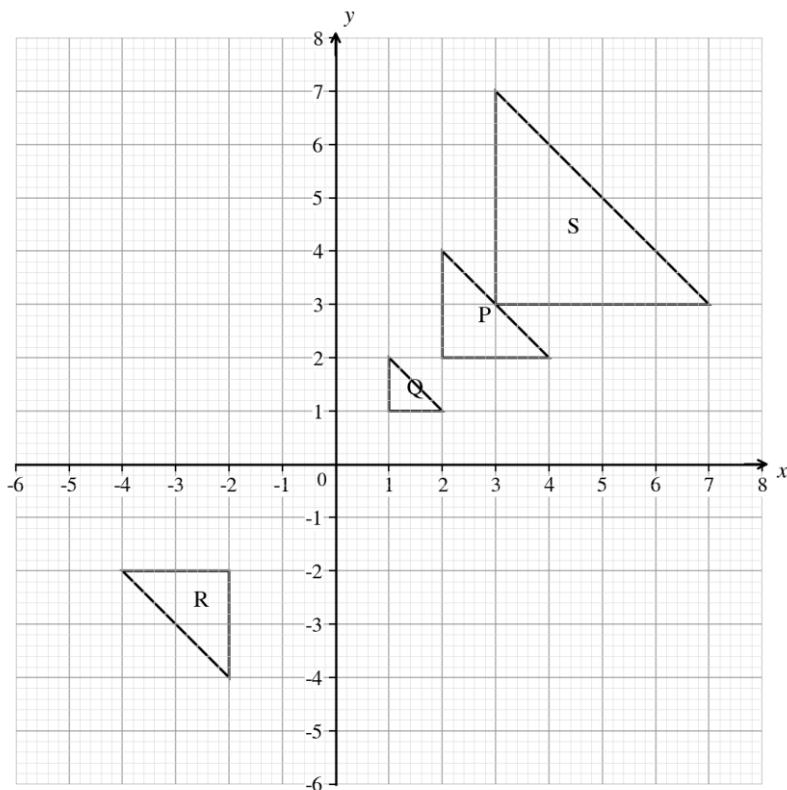
Question 53

(a) $Q: (1, 1), (2, 1), (1, 2)$.

(b) $R: (-2, -2), (-4, -2), (-2, -4)$.

(c) Use $C + 2(P - C): S: (3, 3), (7, 3), (3, 7)$.

(d) Scale factor $\frac{1}{2}$, centre $(0, 0)$. Lines joining corresponding vertices meet at the origin.



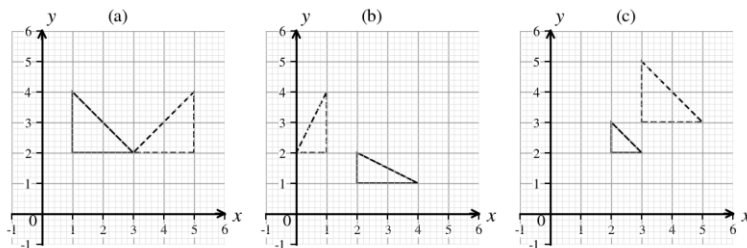
Question 54

(a) Reflection in $x = 3$: midpoints of corresponding pairs lie on $x = 3$.

(b) Rotation 90° anticlockwise about $(1, 1)$. Subtract $(1, 1)$, apply $(u, v) \rightarrow (-v, u)$, then add $(1, 1)$.

(c) Enlargement scale factor 2, centre $(1, 1)$. $C = 2P - P'$; using $(2, 2) \rightarrow (3, 3)$ gives $(1, 1)$, and the other pairs agree.

Solid outline original dashed outline image

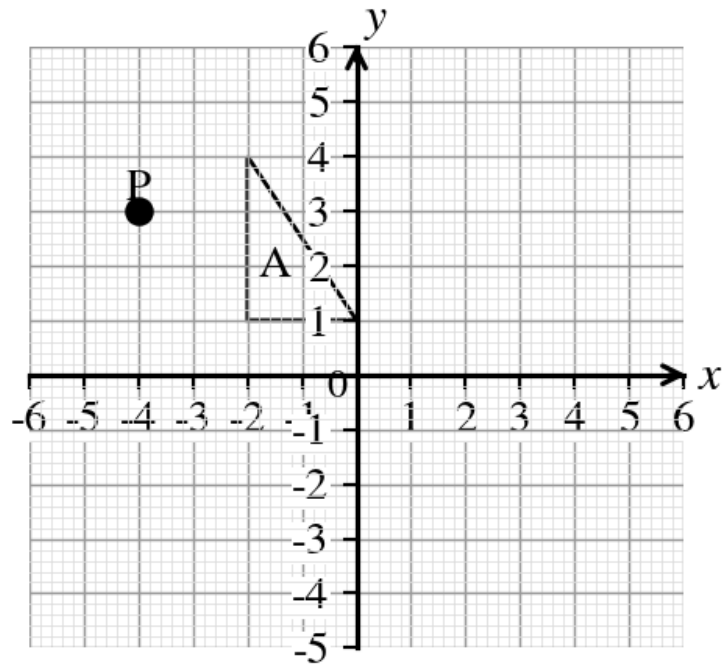


Question 55

(a) $(-4, 3) \rightarrow (4, 3) \rightarrow (4, -3)$.

(b) Rotation 180° about the origin: $(x, y) \rightarrow (-x, -y)$.

(c) Net vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Images: $(0, 4), (2, 4), (0, 7)$.



Statistics

Question 56

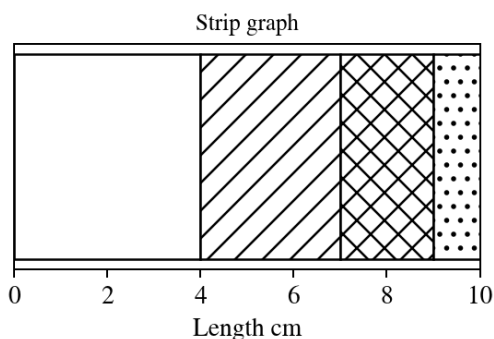
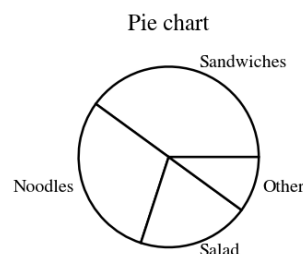
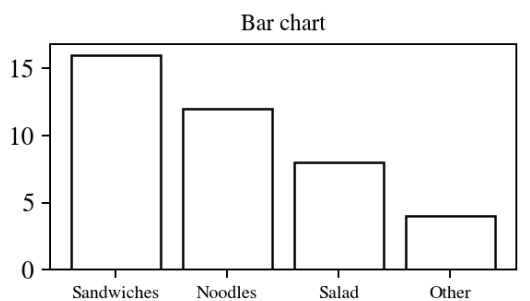
- (a) It underrepresents walkers, cyclists and bus users; travel times may differ.
- (b) Assign each student a unique number; randomly select 30 different numbers, giving each student an equal chance.
- (c) Randomly select 10 students from each equally sized class.
- (d) Choose a random starting point and then every k th name. A repeating pattern in the list can bias selection.
- (e) For example: "How many minutes did your journey to school take today?" Record one response per sampled student. Leading wording influences answers; repeated responses distort counts.
- (f) Raw times vary; record minutes and one value per classmate. Five nearby classmates form a small convenience sample that can miss important groups.

Question 57

- (a) Frequencies for 0, 1, 2, 3, 4 pets: 4, 7, 6, 2, 1. Tallies correspond to these counts.
- (b) Mode 1. Positions 10 and 11 are both 1, so median 1. Total pets = $0 \times 4 + 1 \times 7 + 2 \times 6 + 3 \times 2 + 4 \times 1 = 29$; mean = $\frac{29}{20} = 1.45$. Range = $4 - 0 = 4$.
- (c) Number of pets is discrete; separate bars represent distinct counts.

Question 58

- (a) Symbol counts: 4, 3, 2, 1; key one symbol = 4 students.
- (b) Four separate bars with heights 16, 12, 8, 4.
- (c) Sections 4 cm, 3 cm, 2 cm, 1 cm respectively.
- (d) Angles = frequency/40 $\times$ 360: 144°, 108°, 72°, 36°.
- (e) Sandwiches; $\frac{16}{40} \times 100 = 40\%$. See model displays.



Pictograph one circle = 4 students

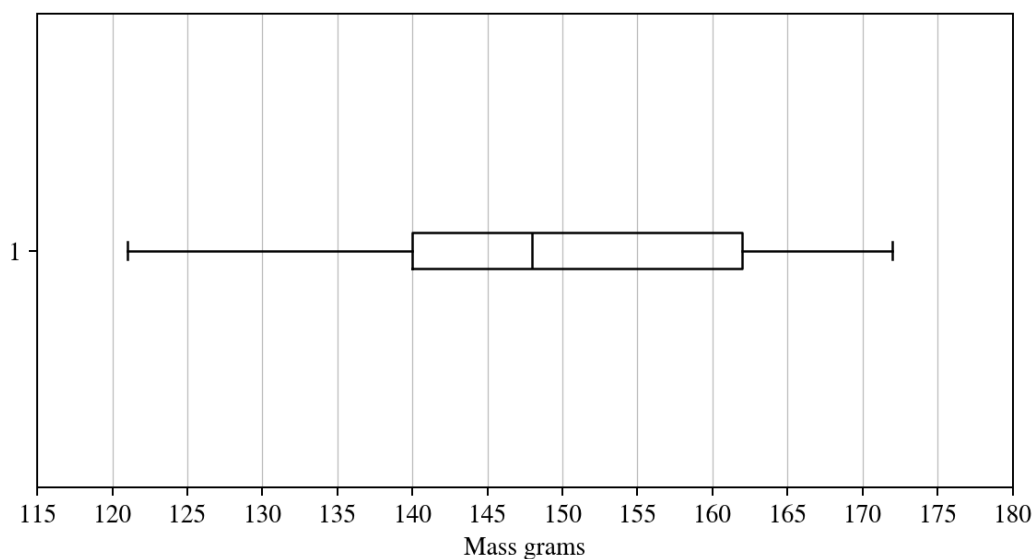
Sandwiches ○ ○ ○ ○
 Noodles ○ ○ ○
 Salad ○ ○
 Other ○

Question 59

(a) Stems/leaves: 12 | 1; 13 | 4 8; 14 | 2 5 5; 15 | 1 6; 16 | 0 4 8; 17 | 2. Key 14 | 2 means 142 g.

(b) Median = $\frac{145 + 151}{2} = 148$ g. $Q_1 = \frac{138 + 142}{2} = 140$ g. $Q_3 = \frac{160 + 164}{2} = 162$ g. $IQR = 22$ g; range = 51 g.

(c) Five number summary (121, 140, 148, 162, 172). Middle half spans 22 g; typical mass around 148 g.

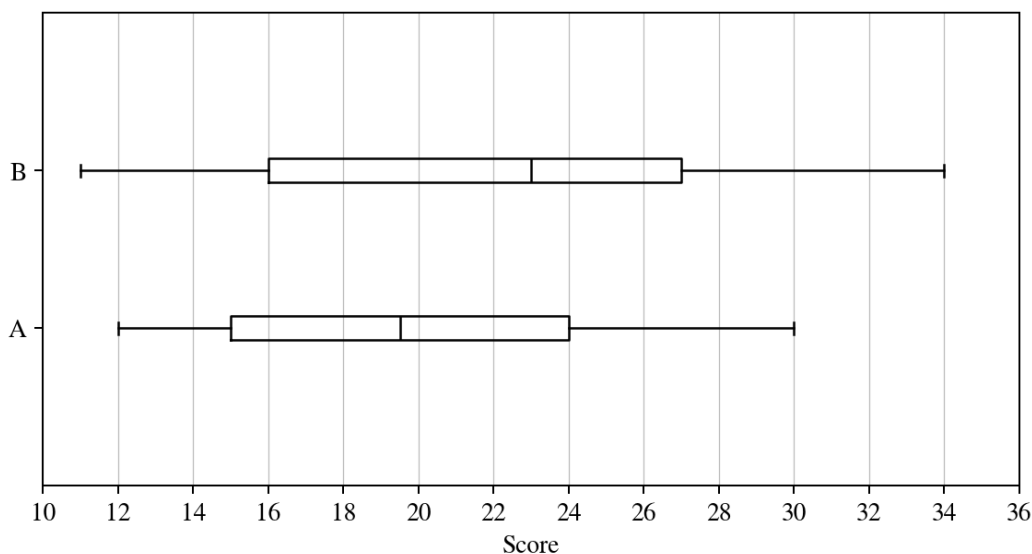


Question 60

(a) See the ordered model diagram. On the left side, leaves increase away from the stem.

(b) A: minimum 12, Q_1 15, median 19.5, Q_3 24, maximum 30. B: minimum 11, Q_1 16, median 23, Q_3 27, maximum 34.

(c) B has a higher typical score (median 23 vs 19.5) and more variable middle half (IQR 11 vs 9).



Back to back stem and leaf model

Group A leaves	Stem	Group B leaves
8 7 5 4 2	1	1 3 6 9
6 4 3 1	2	2 4 5 7 9
0	3	4

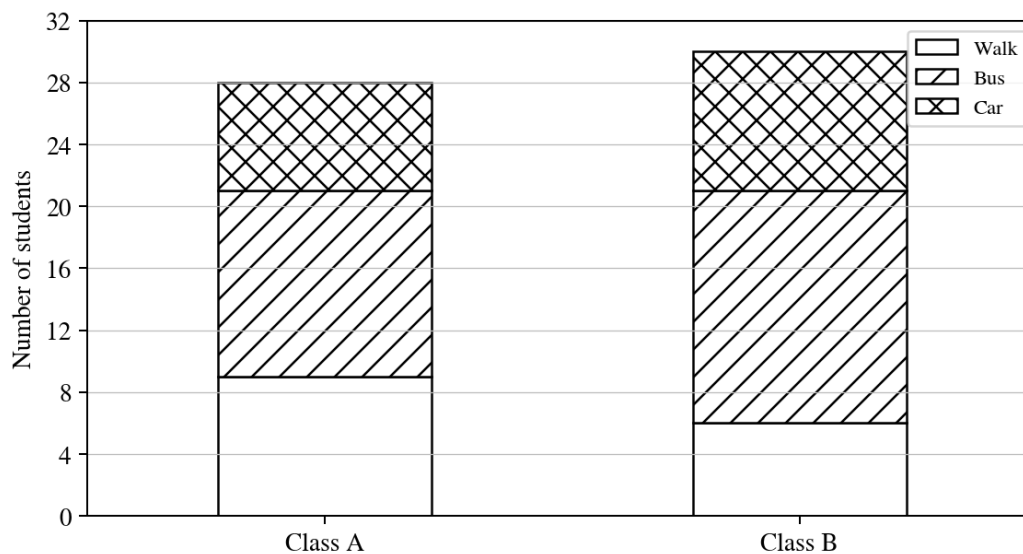
Key: left 2 | 1 means 12 in Group A; right 1 | 1 means 11 in Group B.

Question 61

(a) A stacked segments 9, 12, 7, total 28. B segments 6, 15, 9, total 30; use a consistent key.

(b) $A \frac{12}{28} = 42.9\%$; $B \frac{15}{30} = 50\%$. B has larger proportion.

(c) Travel method alone does not measure distance. Route availability or family choices could explain the difference.



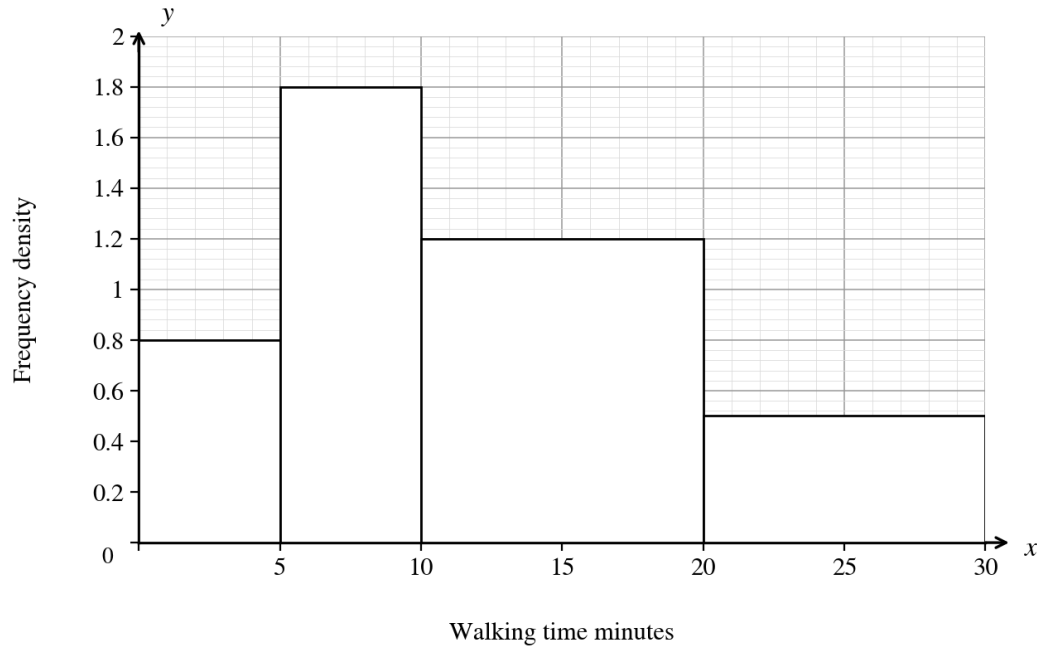
Question 62

(a) Time can take any value within an interval, including fractional minutes.

(b) Density = frequency/class width. Densities 0.8, 1.8, 1.2, 0.5 students per minute. Histogram bars touch and areas represent frequencies.

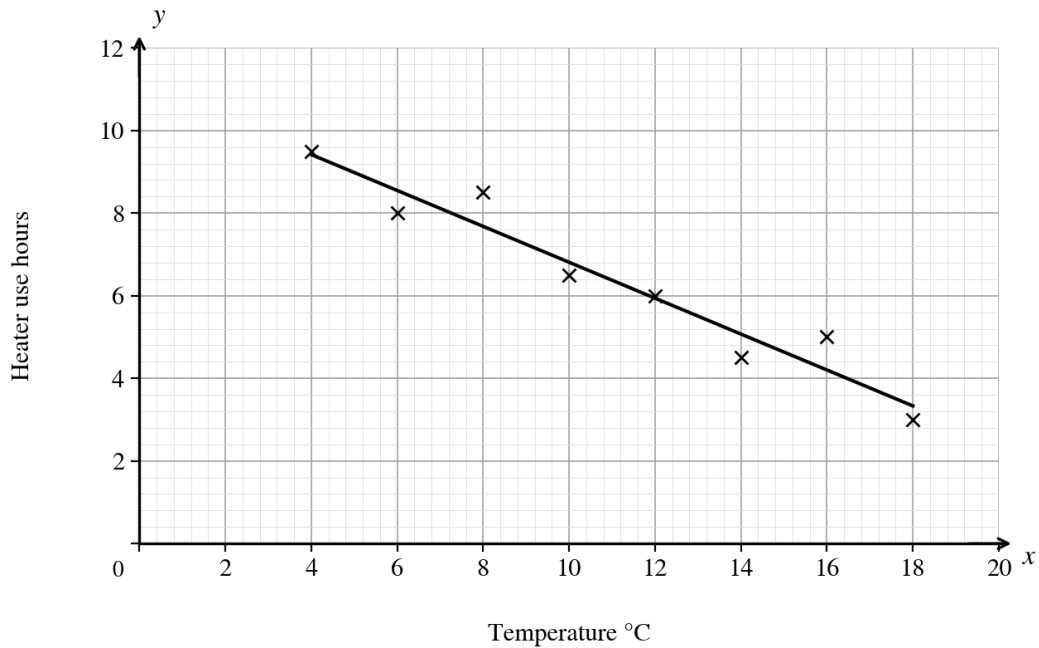
(c) $\frac{12 + 5}{30} = \frac{17}{30} \approx 56.7\%$.

(d) Unequal widths require frequency density; using frequency as height makes areas misrepresent counts.



Question 63

- (a) See scatterplot.
- (b) Strong negative association: use generally decreases as temperature increases.
- (c) A reasonable hand drawn line gives about 6.4 hours (accept sensible estimates, *e.g.* 6.0–6.8). Least squares example: $h \approx 11.15 - 0.435T$.
- (d) 32°C is far outside $4\text{--}18^{\circ}\text{C}$; extrapolation may not follow the observed trend.
- (e) No; insulation, occupancy and other variables may affect use. Association does not establish a sole cause.



Probability

Question 64

(a) $(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T)$. Eight equally likely outcomes.

(b) $(2, H)$ and $(4, H)$: $\frac{2}{8} = \frac{1}{4}$; also $(\frac{1}{2})(\frac{1}{2})$.

(c) $\frac{4}{8} = \frac{1}{2}$; a single roll cannot be both 1 and 4.

(d) Mutually exclusive events cannot both occur; independent events do not change each other's probabilities. The coin and die are independent, while "roll 1" and "roll 4" are exclusive.

Question 65

(a) Roll the same die 120 times on a flat surface, using the same agreed method; record every result in a tally table, without rerolling unwanted outcomes.

(b) Relative frequency = $\frac{43}{120} \approx 0.358$.

(c) Theoretical = $\frac{2}{6} = \frac{1}{3} \approx 0.333$. Small differences can occur by chance.

(d) $600 \times \frac{43}{120} = 215$. Random variation means the actual count may differ.

(e) Results vary. Count your results above 4, divide by 30, and compare with $\frac{1}{3}$; a small sample may differ appreciably.

Question 66

(a) First branches on time 0.8 and late 0.2. On time branches satisfied 0.9, not satisfied 0.1; late branches satisfied 0.35, not satisfied 0.65.

(b) $0.8 \times 0.9 = 0.72$.

(c) $0.8 \times 0.9 + 0.2 \times 0.35 = 0.72 + 0.07 = 0.79$.

(d) $500 \times 0.79 = 395$ customers.

Question 67

(a) First $Y: \frac{4}{7}, P: \frac{3}{7}$. After $Y: Y: \frac{3}{6}, P: \frac{3}{6}$. After $P: Y: \frac{4}{6}, P: \frac{2}{6}$.

(b) $(\frac{4}{7})(\frac{3}{6}) = \frac{2}{7}$.

(c) $(\frac{4}{7})(\frac{3}{6}) + (\frac{3}{7})(\frac{4}{6}) = \frac{4}{7}$.

(d) $1 - P(YY) = 1 - \frac{2}{7} = \frac{5}{7}$.

Question 68

(a) Rows total 30 each. Columns cycle 21, walk 23, bus 16; grand total 60.

(b) $\frac{15}{60} = \frac{1}{4}$.

(c) $\frac{21 + 30 - 12}{60} = \frac{39}{60} = \frac{13}{20}$.

(d) Restrict to 23 walkers: $\frac{15}{23}$.

Sets and Venn Diagrams

Question 69

(a) $A = \{2,4,6,8,10,12,14\}$; $B = \{5,10,15\}$. A only $\{2,4,6,8,12,14\}$; intersection $\{10\}$; B only $\{5,15\}$; outside $\{1,3,7,9,11,13\}$.

(b) $A \cap B = \{10\}$; $A \cup B = \{2,4,5,6,8,10,12,14,15\}$; $A' = \{1,3,5,7,9,11,13,15\}$.

(c) $n(A \cup B) = 9$; n counts the elements of a set.

(d) Shade B outside A ; elements $\{5,15\}$.

Question 70

(a) French only $48 - 19 = 29$; Spanish only $37 - 19 = 18$; both 19; neither $90 - (29 + 19 + 18) = 24$.

(b) 24.

(c) $\frac{29 + 18}{90} = \frac{47}{90}$.

(d) $\frac{19}{48}$.

(e) $S \cap F'$.

Graphing Functions

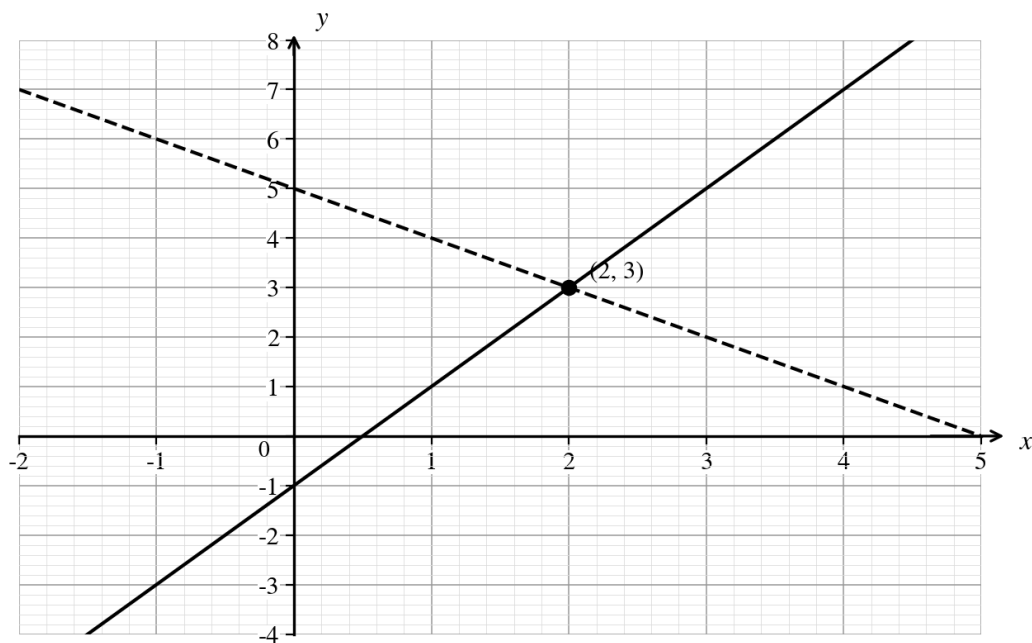
Question 71

(a) First y values $-3, -1, 1, 3, 5, 7$; second values $6, 5, 4, 3, 2, 1$.

(b) See graph.

(c) Intersection $(2, 3)$, so $x = 2$ and $y = 3$.

(d) $2(2) - 3 = 1$ and $2 + 3 = 5$.



Question 72

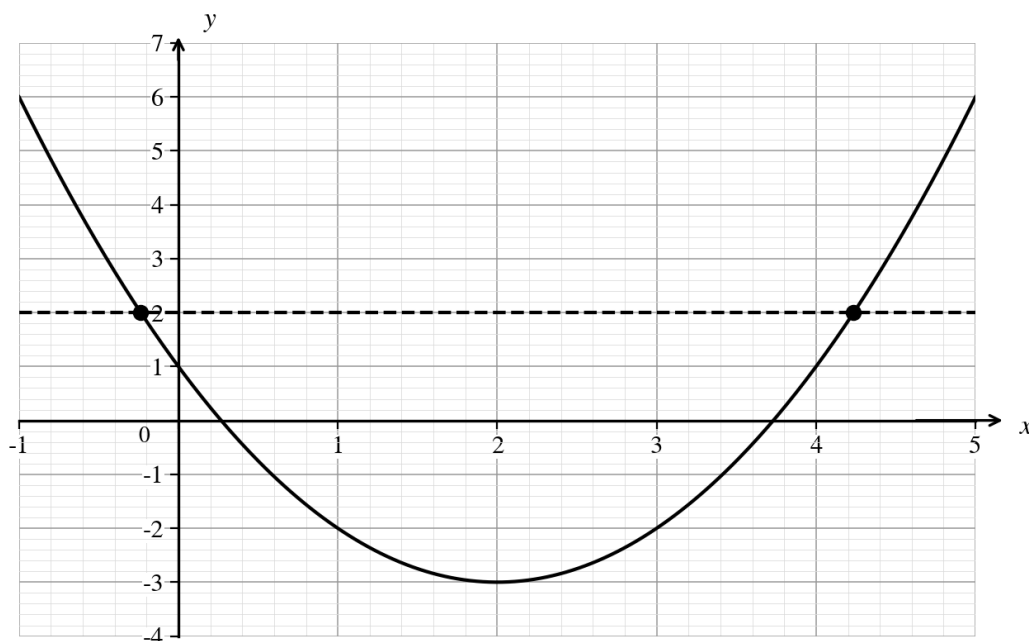
(a) y values 6, 1, -2 , -3 , -2 , 1, 6.

(b) Upward parabola, see graph.

(c) $y = (x - 2)^2 - 3$, so symmetry $x = 2$; turning point $(2, -3)$.

(d) Intersections $x \approx -0.2$ and 4.2 . Algebraic check: $x = 2 \pm \sqrt{5} \approx -0.236, 4.236$.

(e) At each intersection both graphs have the same y value, namely 2; those x values solve the equation.

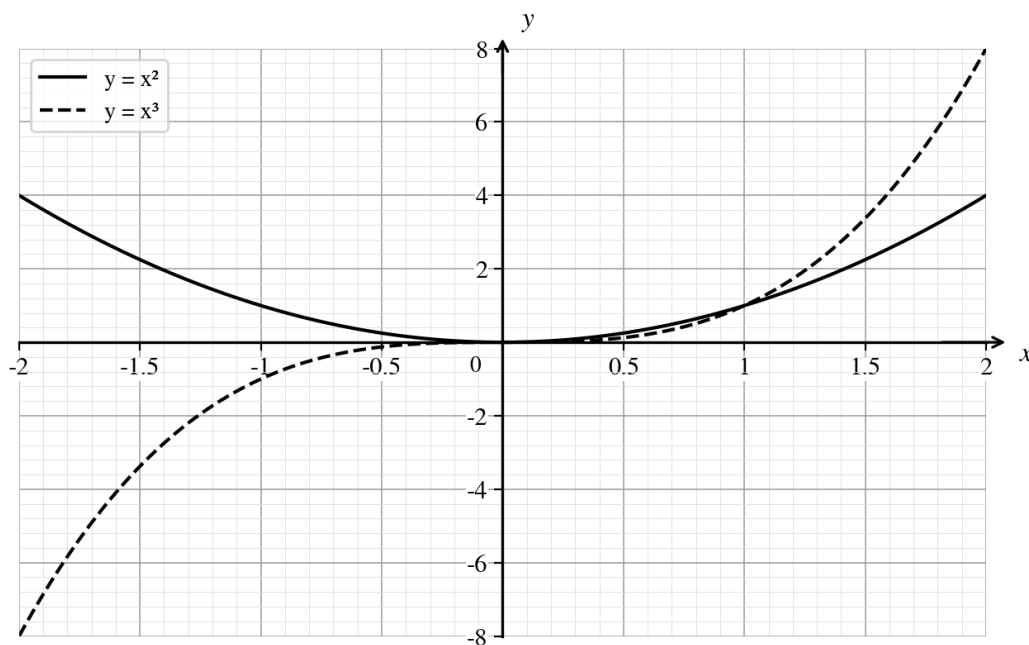
**Question 73**

(a) Square values: 4, 2.25, 1, 0.25, 0, 0.25, 1, 2.25, 4. Cube values: -8 , -3.375 , -1 , -0.125 , 0, 0.125, 1, 3.375, 8.

(b) $y = x^2$ has reflection symmetry in the y axis. $y = x^3$ has rotational symmetry of order 2 about the origin.

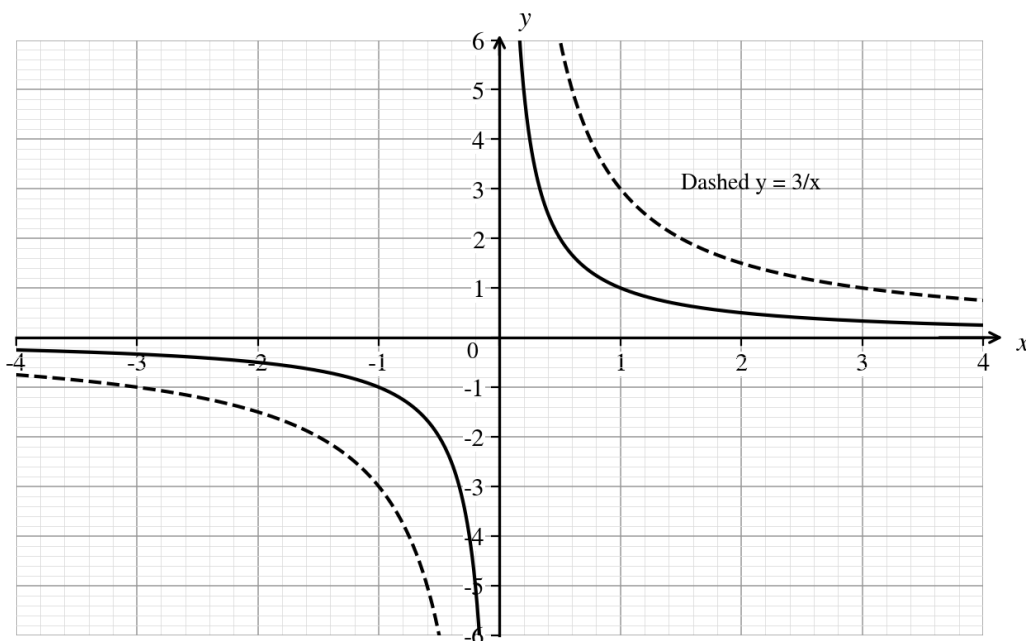
(c) $(0,0)$ and $(1,1)$; $x^2 = x^3$ gives $x^2(1 - x) = 0$.

(d) The cubic $y = x^3$.

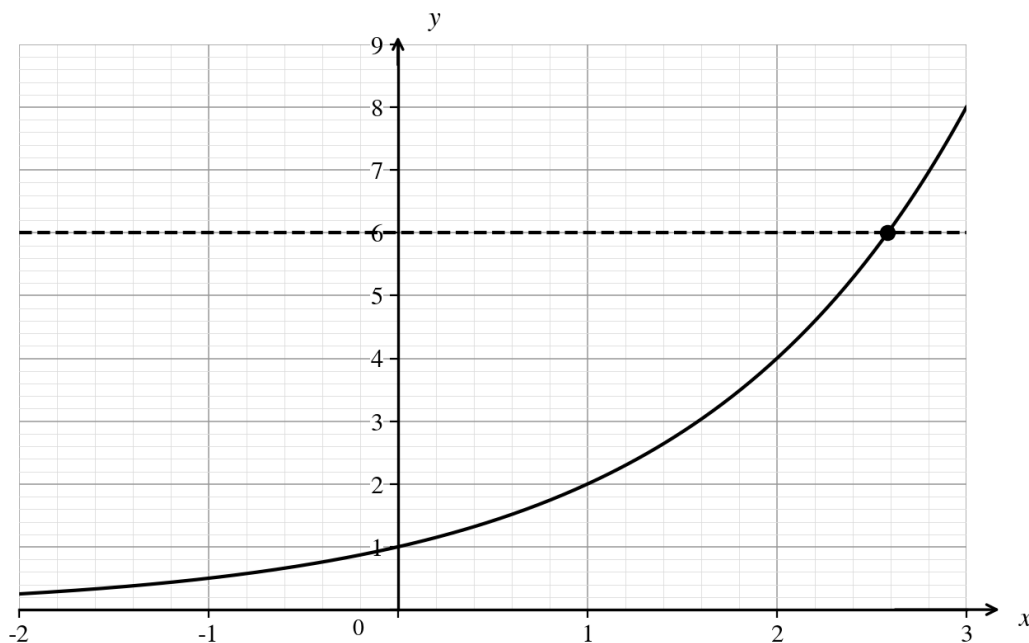


Question 74

- (a) y values $-0.25, -0.5, -1, -2, 2, 1, 0.5, 0.25$. See graph.
- (b) Division by zero is undefined.
- (c) $x = 0$ and $y = 0$. The graph approaches but does not meet these lines; the function has no value at $x = 0$.
- (d) Multiply every y value by 3: a vertical stretch by factor 3; same asymptotes.

**Question 75**

- (a) 0.25, 0.5, 1, 2, 4, 8.
- (b) Intercept (0,1); smooth increasing curve.
- (c) $N = 300 \times 2^t$; $N(5) = 300 \times 32 = 9600$.
- (d) $300 \times 2^t = 1800$ means $2^t = 6$. Read x where $y = 6$: $t \approx 2.6$ hours (actual 2.585...).



Supplementary Proportionality

Question 76

(a) $C = kL$; $k = \frac{28}{3.5} = 8$. Thus $C = 8L$ and $C(8.2) = \$65.60$.

(b) $T = \frac{k}{n}$; $k = 6 \times 15 = 90$. $T = \frac{90}{n}$; $T(10) = 9$ hours. Assume equal productivity, a fixed job and no loss of efficiency.

Question 77

(a) Unit price $= \frac{45}{250} = \$0.18$. Cost $= 1400 \times 0.18 = \$252$.

(b) One printer requires $40 \times \frac{1400}{500} = 112$ minutes; four require $\frac{112}{4} = 28$ minutes.

(c) Cost $= 252 \times 0.90 = \$226.80$; per leaflet $= \frac{226.80}{1400} = \$0.162$, or 16.2 cents.

Supplementary Travel and Conversion Graphs

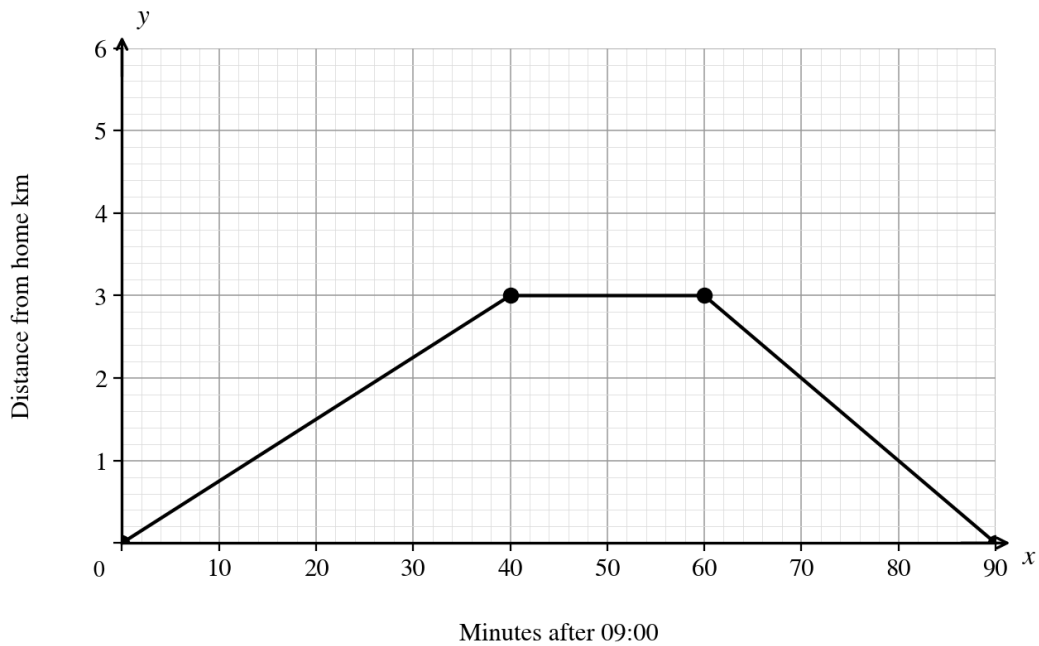
Question 78

(a) Join $(0 \text{ min}, 0 \text{ km})$, $(40, 3)$, $(60, 3)$, $(90, 0)$ with straight segments.

(b) Outward = $\frac{3}{\frac{40}{60}} = 4.5 \frac{\text{km}}{\text{h}}$; return = $\frac{3}{\frac{30}{60}} = 6 \frac{\text{km}}{\text{h}}$.

(c) Total distance 6 km; elapsed 1.5 h; average = $\frac{6}{1.5} = 4 \frac{\text{km}}{\text{h}}$.

(d) Horizontal means stationary; downward means moving towards home, not negative speed.



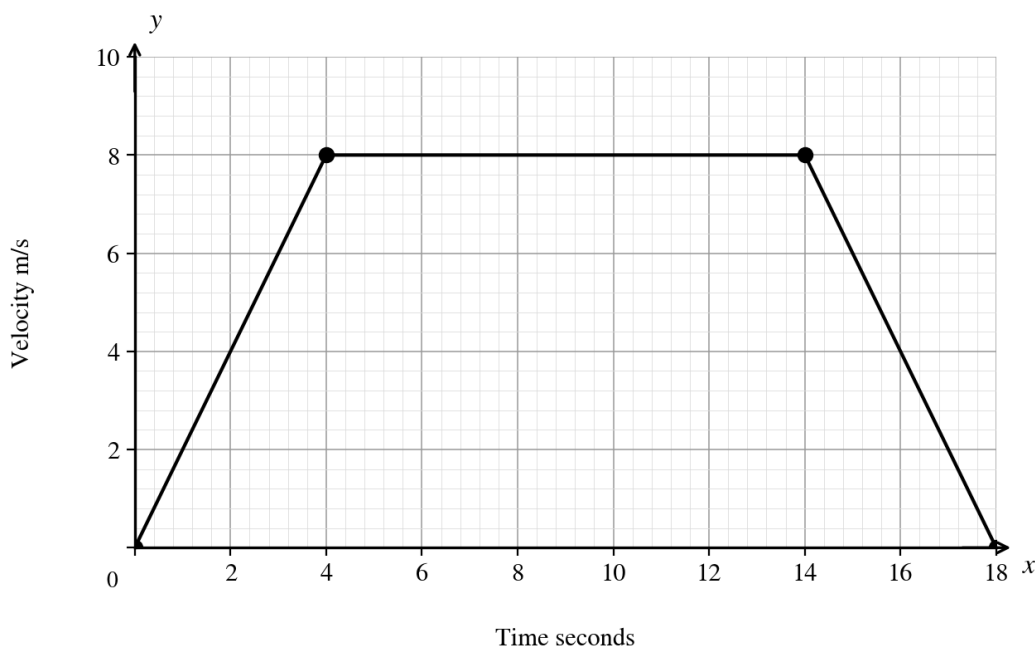
Question 79

(a) Join $(0,0)$, $(4,8)$, $(14,8)$, $(18,0)$.

(b) Gradient = $\frac{8-0}{4-0} = 2 \frac{\text{m}}{\text{s}^2}$.

(c) Area = $\frac{1}{2}(4)(8) + (10)(8) + \frac{1}{2}(4)(8) = 112 \text{ m}$. Velocity is nonnegative, so this is distance.

(d) $\frac{112}{18} = 6.22 \frac{\text{m}}{\text{s}}$ to 3 significant figures.



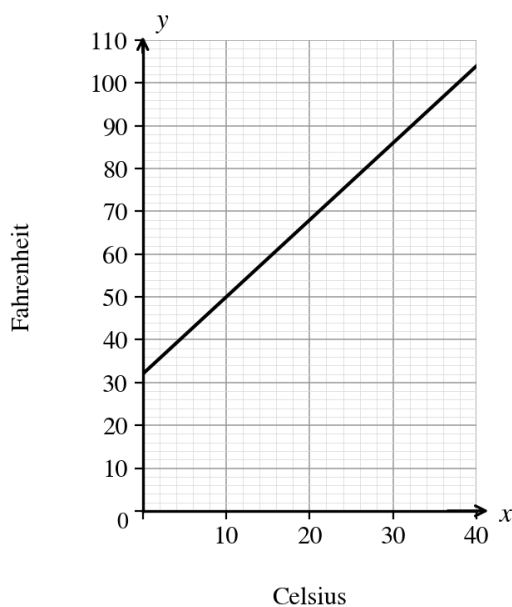
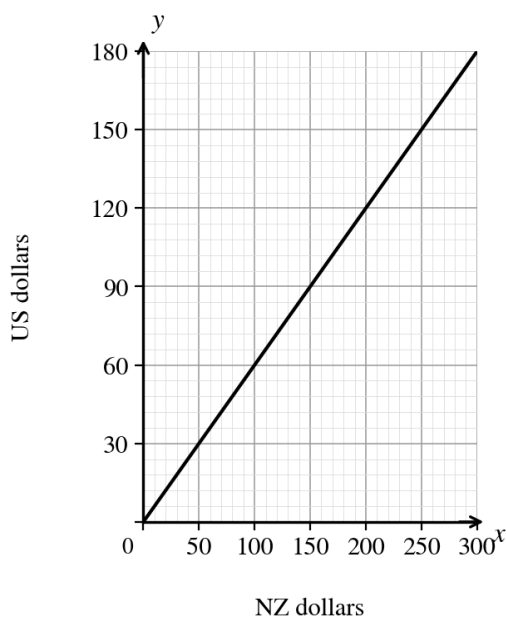
Question 80

(a) $U = 0.60N$, a straight line through $(0,0)$ and $(300,180)$.

(b) $NZ\$180 \rightarrow US\$108$; $US\$135 \rightarrow NZ\$225$.

(c) Plot $(0,32)$, $(10,50)$, $(20,68)$, $(30,86)$, $(40,104)$.

(d) $F = 1.8(25) + 32 = 77^\circ F$; $C = \frac{86-32}{1.8} = 30^\circ C$.



Supplementary Extension Algebra

Question 81

(a) $\frac{2(x+2)+3(x-1)}{(x-1)(x+2)} = \frac{5x+1}{(x-1)(x+2)}$; $x \neq 1, -2$.

(b) $[(x+4)(x+1) - 2(x-2)] \frac{1}{(x-2)(x+1)} = \frac{x^2+3x+8}{(x-2)(x+1)}$; $x \neq 2, -1$.

(c) Factor: $[(x-3) \frac{x+3}{(x-3)(x+2)}] \times [\frac{x-3}{x+3}] = \frac{x-3}{x+2}$; original restrictions $x \neq 3, -2, -3$.

Question 82

(a) $x \neq 2, -1$. Cross multiply: $3x + 3 = 2x - 4$; $x = -7$, allowed.

(b) $x \neq 0, -2$. Multiply by $4x(x+2)$: $4(x+2) + 4x = 3x(x+2)$. Thus $3x^2 - 2x - 8 = 0$; $(3x+4)(x-2) = 0$. $x = -\frac{4}{3}$ or 2 , both allowed.

(c) $w(w+3) = 70$; $w^2 + 3w - 70 = (w+10)(w-7) = 0$. Reject -10 . Width 7 m, length 10 m; perimeter 34 m.

Skill Coverage Guide

Use this guide to locate practice for each learning outcome. Some questions cover several outcomes. The supplementary sections are scheduled after the exams in the assessment schedule.

Number Skills

Signed numbers and decimals 1; fractions and mixed numbers 2; fraction of a quantity and practical fractions 3; simplifying equivalent and sharing ratios 4; rates 5; standard form and calculator arithmetic 6; multi step calculations 3–6. Prime factors are also included in 6.

Percentages

Percentage conversions, percentage of a quantity and comparisons 7; markup, *GST*, increase and decrease 8; reverse percentages 9; simple and compound interest 10; mixed discounts, *GST* and financial arithmetic 11.

Algebra One and Sequences

Reasons for index laws and simplifying powers 12; collecting terms, order of operations and substitution 13; generating and justifying linear patterns 14; quadratic, cubic and linear sequence rules 15; mixed perimeter and area formulae 16.

Pythagoras and Trigonometry

Side labels, ratio selection, Pythagoras and inverse trigonometry 17; unknown shorter side and calculator trig values 18; trig sides including hypotenuse 19; practical combined problem and rounding 20; three dimensional Pythagoras and trigonometry 21.

Mensuration

Metric lengths, area and volume conversions, capacity and bounds 22; common plane areas, irregular area and polygon perimeter 23; circle and composite perimeter and area with practical rounding 24; plane faced, cylindrical and spherical surface areas 25; all specified solid volumes 26; volume, capacity, percentage, rate and accuracy modelling 27.

Linear Equations and Inequalities

Linear equations and inequality number line 28; changing the subject 29; simultaneous equations 30; practical justified formula and inequality 31; forming and checking simultaneous equations in context 32. Linear n th term also appears in 14–15.

Straight Line Graphs

Integer coordinates, tables, plotting $y = mx + c$, slope, intercept and parallels 33; reading a given line, deriving equations and checking a point 34.

Geometry

Straight line, point, vertically opposite, triangle and quadrilateral angles 35; corresponding, alternate, co interior angles and multi step reasoning 36; polygon interior and exterior angles 37; central angle, same arc, semicircle and cyclic quadrilateral 38; radius tangent, alternate segment and perpendicular chord theorems 39; multi step circle reasoning 40; similar lengths and areas and congruence 41; bearings, reverse bearings, scale drawing and symmetry 42; ruler and compass triangle construction 43.

Algebra Two

One and two bracket expansion and an area model 44; common factor, difference of squares, trinomials and two stage factorising 45; simplifying algebraic fractions 46; quadratic equations 47; mixed equation forming, factorising, perimeter and cost 48.

Vectors

Representation, column vectors, scalar multiples, addition, subtraction, magnitude and direction 49; drawing vector subtraction, connections and unknown components 50; practical resultant, bearing and displacement 51.

Transformations

Translations, reflections and rotations 52; positive, fractional and negative enlargements 53; describing reflection, rotation and enlargement with centres 54; combined transformations and net effect 55.

Statistics

Sampling, randomness, bias and collection design 56; tally, frequency table, mean, median, mode and range 57; pictograph, bar, strip and pie graphs 58; stem and leaf, quartiles, *IQR*, box plot and formal description 59; back to back stems, comparative boxes, centre and spread 60; composite bars and evaluation of conclusions 61; continuous data, histogram and criticism of a display 62; scatterplot, best fit, interpreting association and evaluating extrapolation 63.

Probability

Sample space, theoretical probability, exclusive and independent events 64; experiment design and empirical relative frequency 65; unequal branch trees and combined events 66; without replacement trees and complements 67; multi variate tables and conditional probabilities 68; Venn diagram probabilities 70.

Sets and Venn Diagrams

Set language and notation, elements, union, intersection, complement, diagrams and cardinality 69; practical cardinality and probability 70.

Graphing Functions

Graphical simultaneous equations 71; quadratic graph and graphical roots 72; $y = x^2$ and $y = x^3$ 73; $y = \frac{1}{x}$ and reciprocal graphs 74; exponential graph and contextual application 75.

Supplementary Proportionality

Direct and inverse proportion 76; mixed direct, inverse and percentage problem 77.

Supplementary Travel and Conversion Graphs

Distance time graphs, speed and travel interpretation 78; velocity time graph, gradient and area 79; currency conversion and change of scale 80.

Supplementary Extension Algebra

Simplification and addition and subtraction of algebraic fractions 81; fractional equations, forming expressions, distributive expansion and quadratic factorising 82. Expansion is also practised in 44.